

# **ORBIT UNCERTAINTIES, KEYHOLES, and COLLISION PROBABILITIES**

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# ORBIT UNCERTAINTIES

- All orbital estimates are uncertain, because the observations upon which they are based contain small measurement errors
- Standard assumption: measurement errors are uncorrelated, unbiased, and Gaussian (standard deviation is typically 1 arc-sec)
- A by-product of Gaussian Least Squares is the orbit covariance matrix, which quantifies the orbit uncertainty (basically 6 standard deviations and associated error correlations)
- At a given confidence level, say  $\sigma = 3$ , the orbital uncertainty is represented by a six-dimensional hyper-ellipsoid in orbital element space
- Essentially, the hyper-ellipsoid bounds the region of acceptable orbits (i.e., orbits consistent with the observations)

# COMPUTING CLOSE-APPROACH UNCERTAINTIES

- We use two basic methods:
  - **Linear Covariance Method**
    - semi-analytic, elegant and fast
    - not applicable when uncertainties become large
    - used for our analysis of 1997 XF11
  - **Nonlinear Monte Carlo Method**
    - requires much more computation
    - allows prediction of impact probability much farther into the future
    - used for our analyses of 1999 AN10 and 1998 OX4

# IMPACT PLANE

- At each close approach, compute the direction of the incoming asymptote of the planet-relative hyperbolic trajectory.
- The *impact plane* (or b-plane) is the plane perpendicular to the incoming asymptote.
  - Use of the impact plane reduces the nonlinear effects due to gravitational focusing.
- Change the impact-plane scale so that the capture radius becomes equal to the radius of the planet.
- The planet-relative position of the asymptote intercept is the scaled b-vector  $\mathbf{b}_p$ . If  $\mathbf{b}_p < r_p$ , a collision will occur.

## LINEAR COVARIANCE METHOD

- Compute linear mapping from variations in orbital elements at epoch to variations in  $\mathbf{b}_p$ .
- Map orbit element uncertainties at epoch to obtain uncertainty in  $\mathbf{b}_p$ , which is an ellipse (or, more precisely, a 2D probability density).
- Compute impact probability by integrating the 2D probability density over a circle of radius  $r_p$ , which represents the figure of Earth.

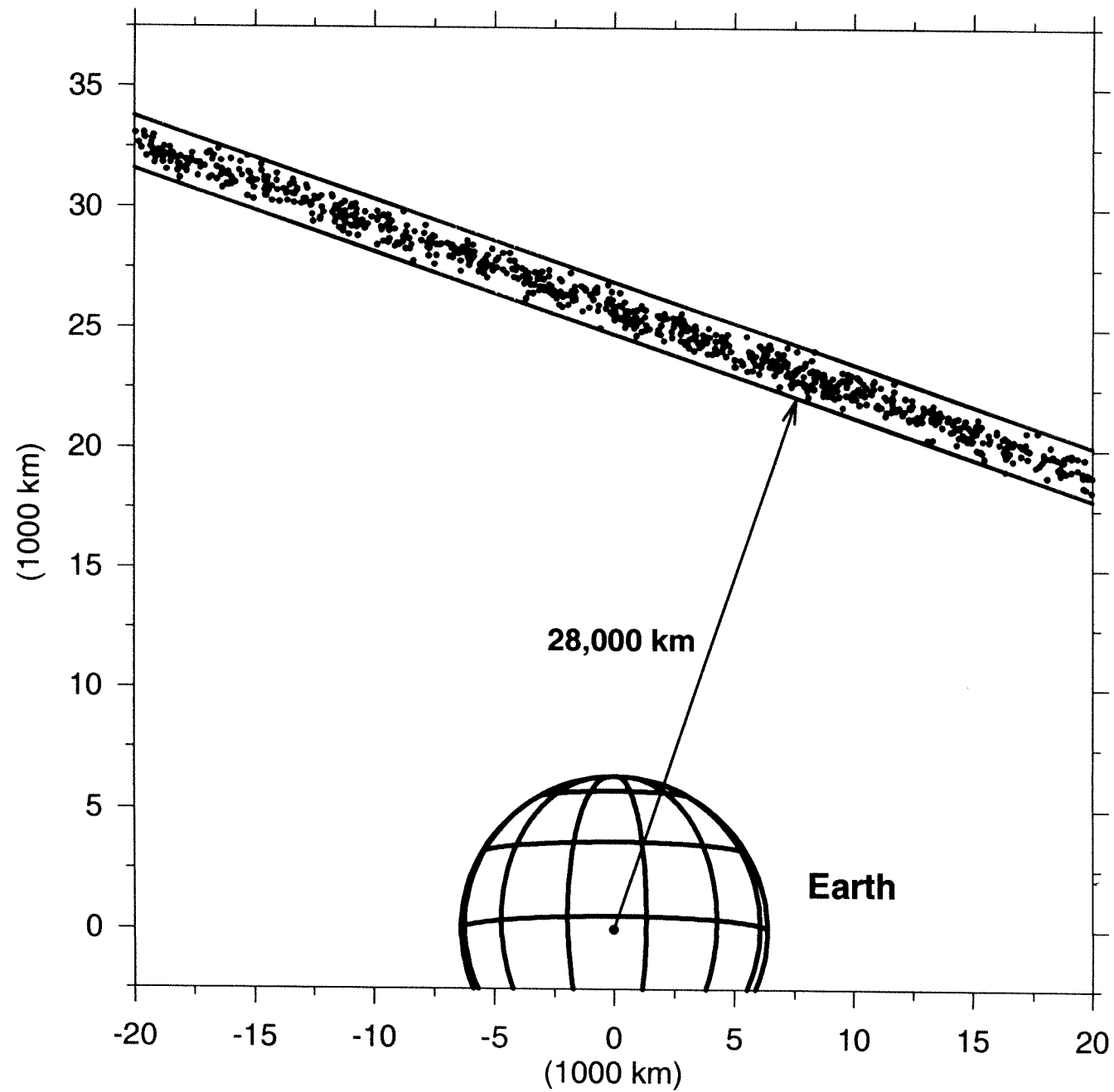
## NON-LINEAR MONTE CARLO METHOD

- Populate the 6-dimensional hyper-ellipsoid of initial orbital elements with thousands of random points to obtain an ensemble of possible initial conditions.
- Numerically integrate each orbit forward in time, using the fully nonlinear equations of motion, and compute  $\mathbf{b}_p$ .
- Plot all the  $\mathbf{b}_p$  points in a common impact plane.

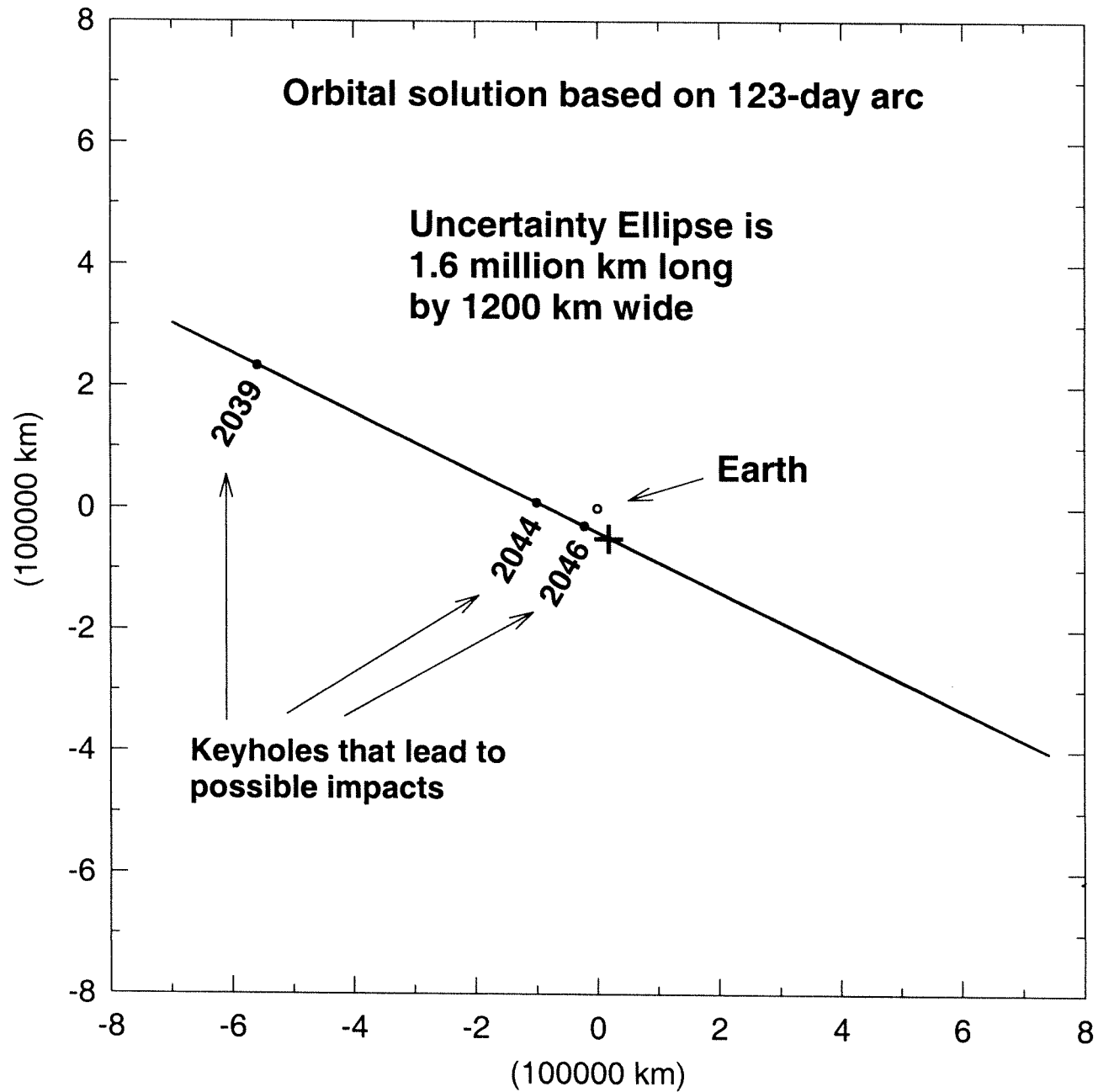
## KEYHOLES AND STREAMS

- The points in the impact plane will fall either on the *main stream* of the orbital uncertainty or on *sub-streams*, which are generally created by earlier passage through *keyholes*.
- A *keyhole* is a narrow linear slice of a confidence region where the encounter perturbs the object onto a trajectory which returns for another close approach on a subsequent orbit.
- There are as many as three keyholes per orbital period commensurability, and there may be several commensurabilities which lead to a given close approach.
- The impact probability of each stream of the confidence region is estimated by computing the average density of points along the line of variation and multiplying by the chord length across the Earth disk.

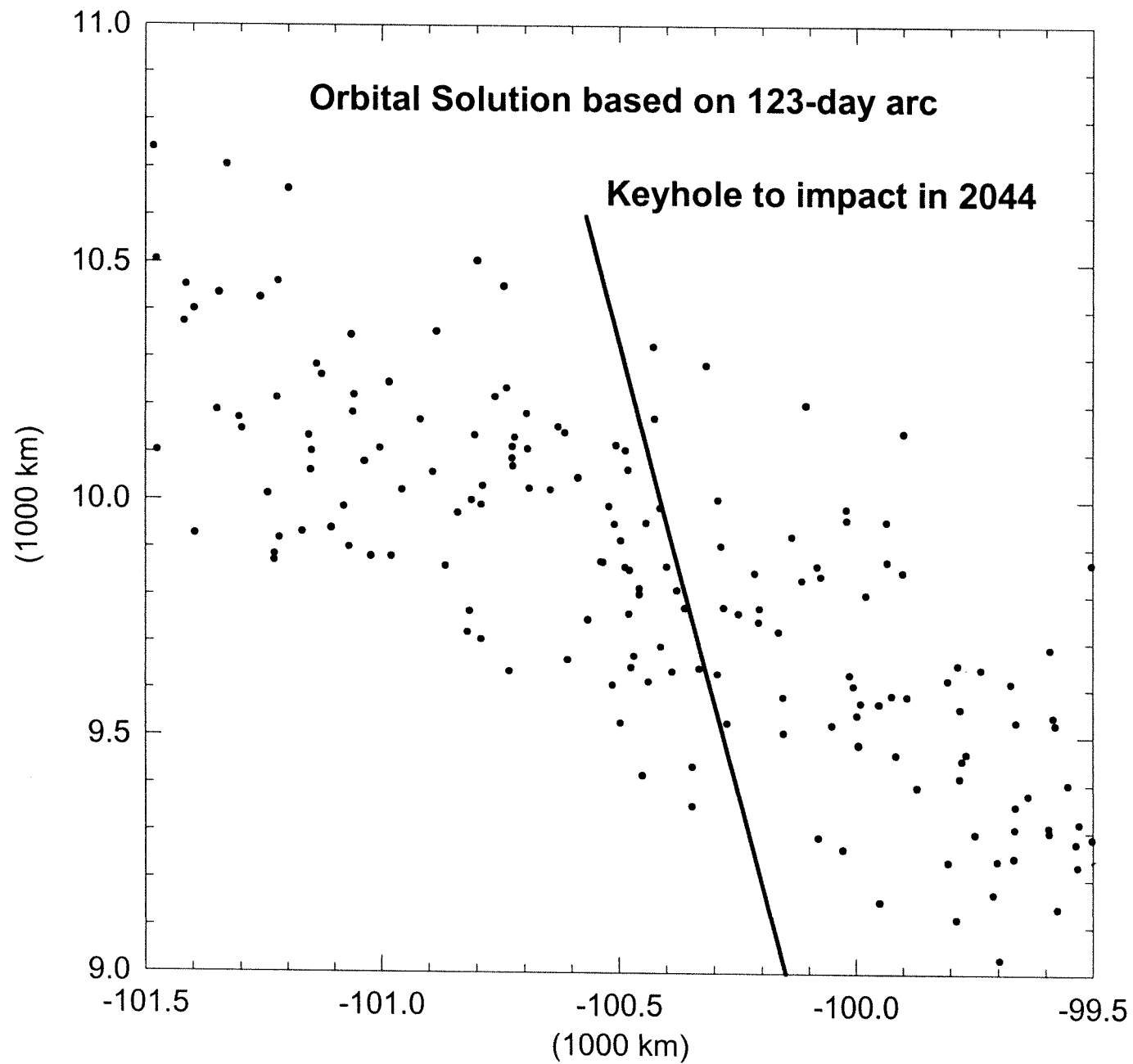
## 1997 XF11: Monte Carlo Points in Earth Impact Plane in 2028



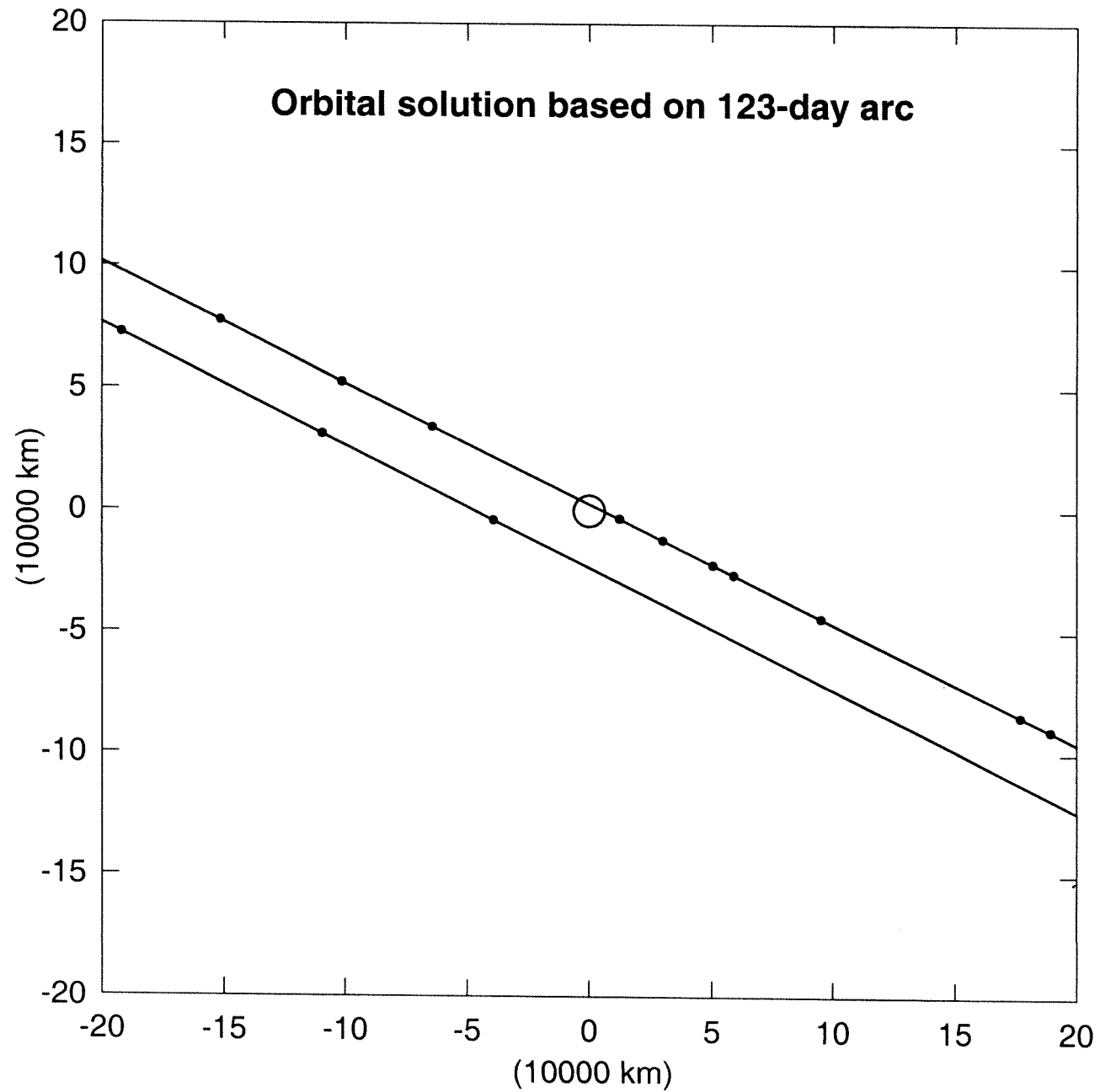
## 1999 AN10: Impact Plane on 2027 Aug 07



# 1999 AN10: Impact Plane on 2027 Aug 07



## 1999 AN10: Impact Plane on 2044 Aug 06



## 1999 AN10: Keyholes in Impact Plane on 2027 Aug 07

